plates, the results for the coarsest grid are not as accurate as for the simply supported plate; however, as the element size is decreased, the values of deflection obtained with the present elements approach the exact results very rapidly. Detailed discussion of results for static and free vibration analysis of square isotropic plates is given in Ref. 11. The values of calculated bending moments and corner reactions are given in Table 1. Good agreement with exact values is observed for practical mesh subdivisions.

The effect of transverse shear in moderately thick isotropic plates can be represented using the present elements. The values of the nondimensional deflection coefficients (Gw/hq) for various h/L ratios for simply supported and clamped plates are compared to the 3-D elasticity solution[13,14] in Table 2. Only $\frac{1}{4}$ plate was analyzed and Q mesh with number of elements per edge equal to 8 was used. Good agreement is seen with the 3-D elasticity solution.

Concluding Remarks

A new triangular plate-bending finite element using a quintic displacement field but having only displacement and rotations as grid point degrees of freedom is described in this Note. The examples presented demonstrate that high accuracy is achievable using this element for practical mesh subdivisions. The effect of transverse shear deformations is included in the element formulation. The present element gives satisfactory approximations for solving isotropic plate problems for cases where transverse shear effects are significant. This element is ideally suited for inclusion into general-purpose computer programs due to 1) high accuracy for practical mesh subdivisions, 2) use of only displacements and rotations as grid point degrees of freedom, and 3) inclusion of transverse shear flexibility in the element properties.

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Suppression of Ionization Instability in an MHD Disk Generator

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STABILIZATION of the ionization instability in a nonequilibrium MHD plasma by fully ionizing the seed has been observed in simulation experiments[1,2] where the current was supplied from the external circuit. Here we present the experimental results of the recovery of the effective Hall parameter in the regime of fully ionized seed in an actual nonequilibrium MHD disk generator.

The experimental conditions of the present work are summarized in Table 1. All measurements were made in the Hall open circuit as a function of the magnetic field. The effective Hall parameter was calculated from the radial electric field. The electron temperature was estimated from the intensity of the potassium resonance line (7699A).

Figure 1 shows the variation of the time-averaged line intensity against magnetic field. We can see that the intensity decreases as the magnetic field increases above 2.5 kg. This

Table 1 Experimental conditions of the present work

Working gas		Disk generator	
Ar + Potassium heated by the pressure driven shock tube		radius of the inner electrode	5 cm
		radius of the outer electrode	11 cm
Conditions in the MHD channel		channel height	
pressure	40 torr	1.97 cm at the inner electrode	
temperature	900 K	1.00 cm at the outer electrode	
velocity	1400 m/sec		
Mach number	2.4		
seed fraction	8×10^{-5}		

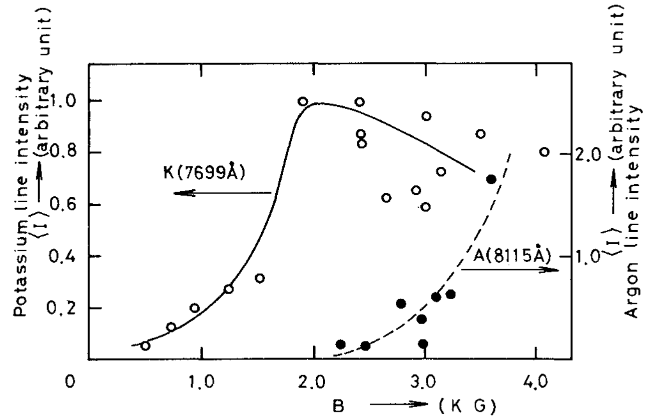

Fig. 1 Intensity of the potassium resonance line (7699Å) and the argon line (8115Å) as a function of magnetic field.

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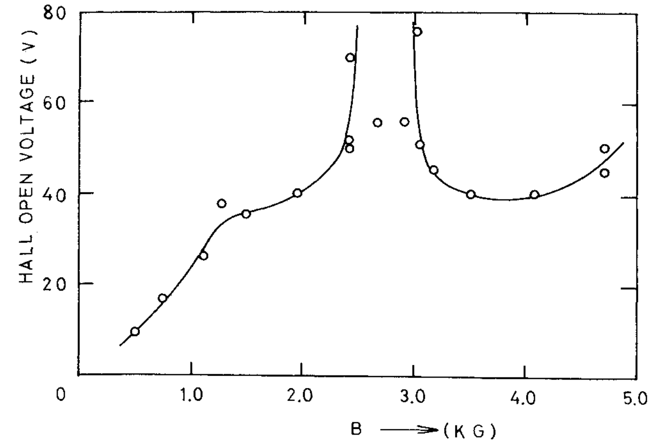

Fig. 2 Hall open voltage as a function of magnetic field.

fact means that the degree of ionization of the seed approaches to unity so that the number density of neutral seed atoms decreases. It can be seen also in Fig. 1 that the argon radiation intensity (8115A) increases for the magnetic field above 2.5 kg.

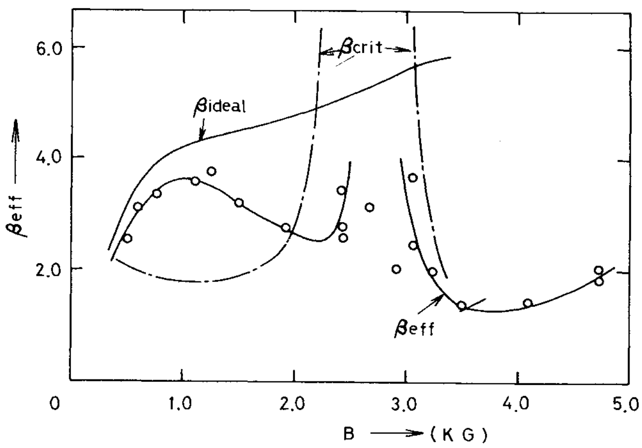

Fig. 3 Effective Hall parameter as a function of magnetic field.

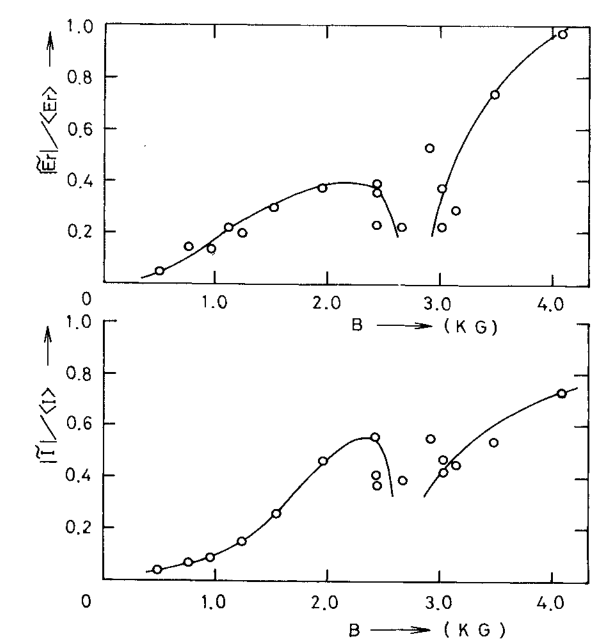

Fig. 4 Fluctuations of the radial electric field and the potassium resonance line as a function of magnetic field.

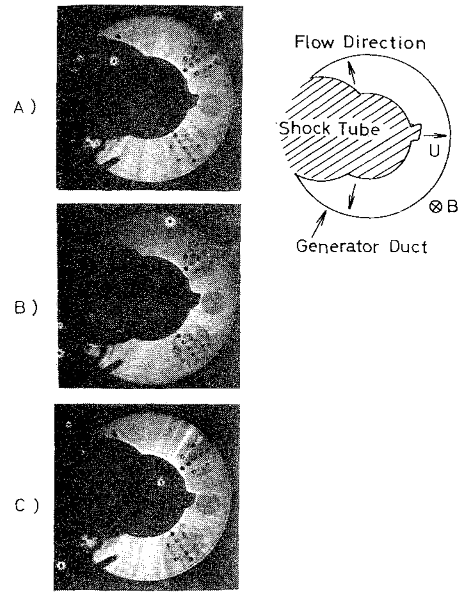

Fig. 5 Photographs of the discharge: a) $B = 1.95$ kg, b) $B = 3.03$ kg, c) $B = 4.70$ kg.

Figure 2 shows the experimental results of the Hall open voltage against the magnetic field. We can see large values of the Hall voltage for the magnetic field between 2.5 kg and 3 kg. To understand this important result more clearly, we plot in Fig. 3 the values of the effective Hall parameter β_{eff} which is estimated from the measured Hall voltage. Figure 3 also shows the ideal Hall parameter β_{ideal} and the critical Hall parameter β_{crit} for the ionization instability. It can be seen from Fig. 3 that β_{eff} decreases when β_{ideal} exceeds β_{crit} and that β_{eff} recovers up to 4 for the magnetic field between 2.5 kg and 3 kg where $\beta_{\text{crit}} > \beta_{\text{ideal}}$. It is also seen in Fig. 3 that β_{eff} decreases as the magnetic field increases above 3 kg where ionization of argon begins so that β_{ideal} again exceeds β_{crit}.

The fluctuations of the radial electric field and the radiation intensity of 7699A are shown in Fig. 4. We can see the reduction of the fluctuations for the magnetic field between 2.5 kg and 3 kg. This fact clearly indicates that the recovery of the effective Hall parameter is due to the reduction of the ionization instability.

Figure 5 shows photographs of the discharge in the disk generator which were taken with the exposure time longer than the experiment time. Figure 5a for the magnetic field of 1.95 kg shows the spoke instability[3] which appears in the disk generator for $\beta_{\text{ideal}} > \beta_{\text{crit}}$. Figure 5b for the magnetic field of 3.03 kg shows the reduction of the spoke instability, and Fig. 5c for the magnetic field of 4.70 kg indicates the spoke instability which grows again as a result of the initiation of the ionization of argon.

In conclusion, we observed the recovery of the effective Hall parameter in the regime of the fully ionized seed. This fact was assured by photographs of the discharge and measurements of the electric field, the resonance line intensity of the seed and their fluctuations.

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